Efficient Neural Clause-Selection Reinforcement

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Machine-Learning-Boosted Automated Theorem Proving

ATP technology:

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Outline

1. Saturation and Clause Selection
2. RL-Inspired Guidance
3. Neural Clause Evaluation
4. Deepire 2.0

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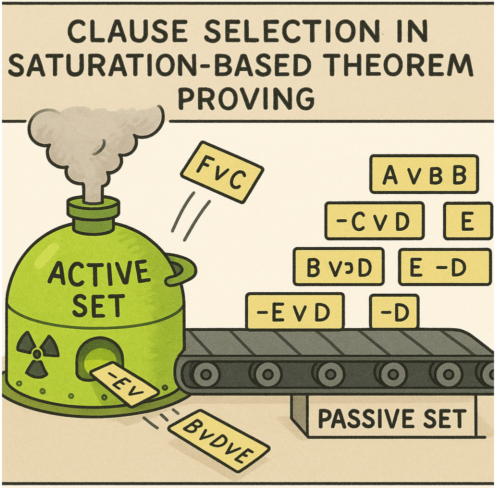
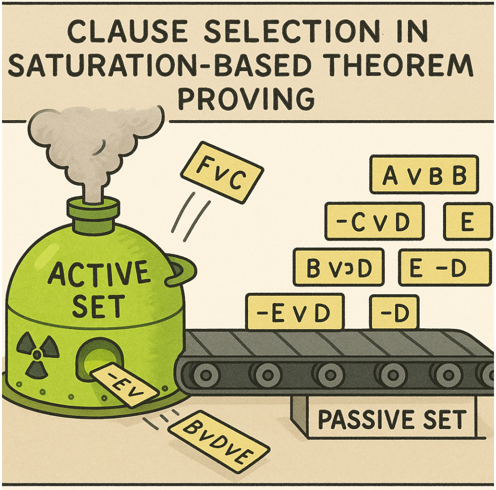
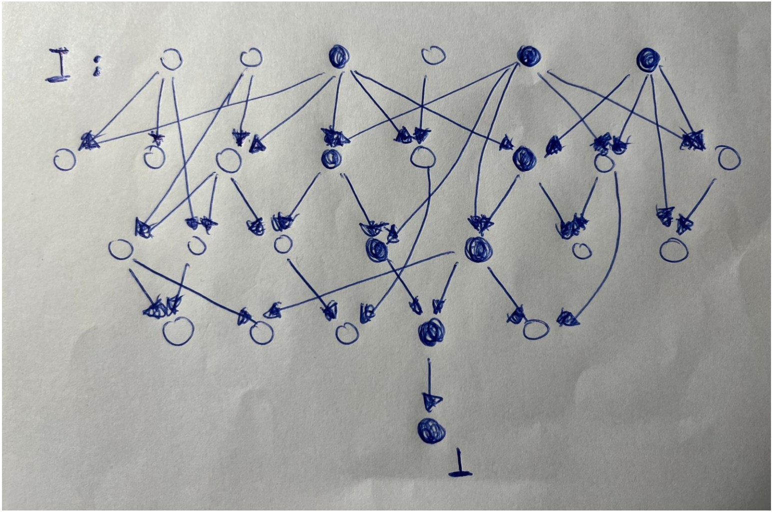
Saturation-based Theorem Proving

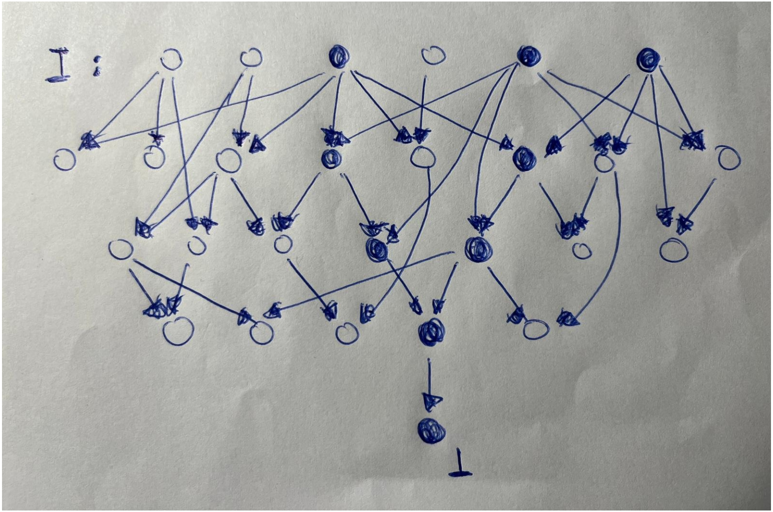




At a typical successful end: $|\mathit{Passive}| \gg |\mathit{Active}| \gg |\mathit{Proof}|$

The Proof Is Often Just A Tiny Part





How close can we actually hope get to the perfect clause selection?

How is clause selection traditionally done?

Take simple clause evaluation criteria:

- age: prefer clauses that were generated long time ago
- weight: prefer clauses with fewer symbols

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- alternate between selecting from the queues using a fixed ratio

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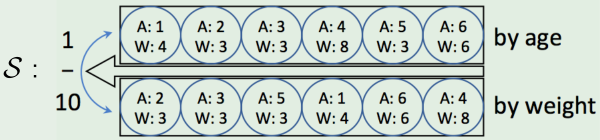
Example (Organizing *Passive* via two priority queues)



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- It programs itself (sometimes even optimally, in the limit)
- It could discover fundamentally novel tricks and hacks!

Key Reinforcement Learning Concepts

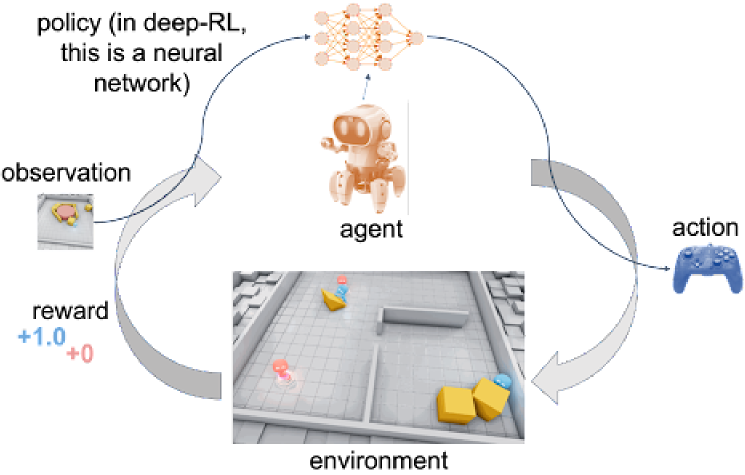

*Illustration from anyscale.com.

Saturation as an Reinforcement-Learning Environment

Agent

- the clause selection heuristic

Action

- the next clause to select <u>from the current passive set</u>

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➥ TRAIL [Crouse et al.'21], [McKeown'23], [Shminke'23], ...

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Reward

- refusing the play the honest, super-sparse reward game
- like in ENIGMA: a proof clause is a good clauses

Towards the RL-Inspired Learning Operator

A trace of a successful proof attempt on problem P is a tuple

$$T = (P, \mathcal{C}, \mathcal{C}^+, \{\mathcal{P}_i\}_{i \in I_T}).$$

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Learning operator (for clause selection)

- input: neural network $N_{\boldsymbol{\theta}}$ (learnable params $\boldsymbol{\theta}$), set of traces $\mathcal{T}$
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$$\pi_{C,\boldsymbol{\theta}} = \mathrm{softmax}_C\left(\{l_D\}_{D \in \mathcal{P}}\right) = \frac{e^{l_C}}{\sum_{D \in \mathcal{P}} e^{l_D}}$$

is the (stochastic) clause selection policy defined by $N_{\boldsymbol{\theta}}$

The RL-Inspired Operator

Policy Gradient Theorem [Williams'92]

To improve a policy in terms of the expected return we update

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Each moment in time i is an independent opportunity to improve, with

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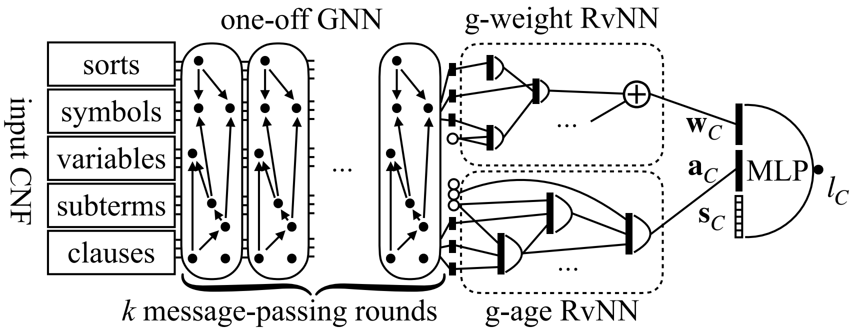
Simple Hand-Crafted Features on Top!

Architecture Diagram



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I still need to try out how much GPUs could help here . . .

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- little trick; despite the RL heritage:
  inner loop trains until validation loss does not improve

Experiments

Setup:

- TPTP v9 CNF+FOF, 19 477 problems (train/test split)
- Vampire's default strategy (1:1 age-weight alternation)
- limit of 30 000 Mi (∼10 s) per proof attempt

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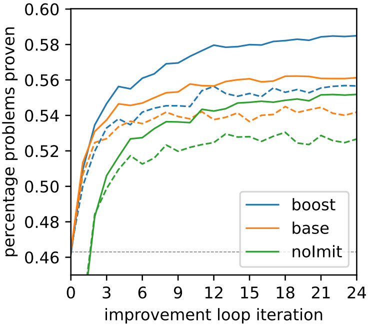

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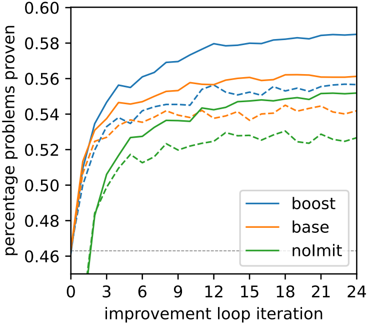

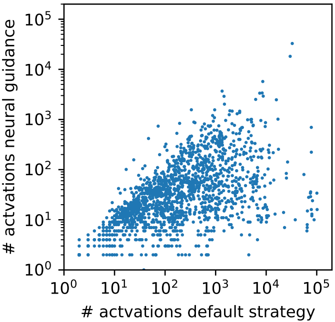

Experiments II

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- overfit to TPTP with 100 000 Mi-limited runs
- ran for 12.4 days
- solved 130 rating 1.0 (49 never solved, 8 status UNK)

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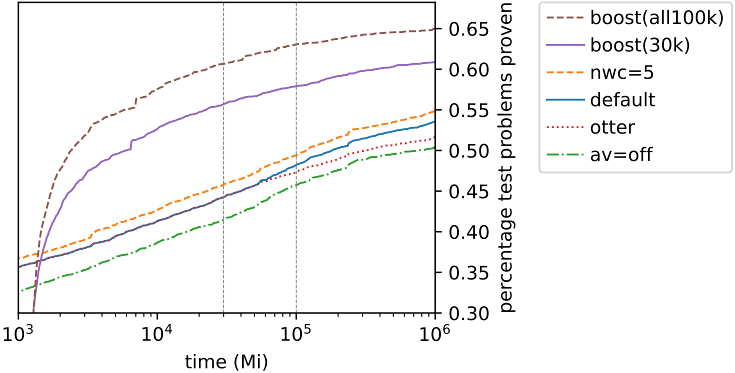

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- other benchmarks than TPTP; e.g. Mizar40; transfer learning
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The "pos/neg"'s of E:

E prover can be asked to output, for every clause selected in a run, whether it ended up in the final proof (pos) or not (neg)

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E prover can be asked to output, for every clause selected in a run, whether it ended up in the final proof (pos) or not (neg)

Next comes the ML:

- represent those clauses somehow (features, NNs, ...)
- train a binary classifier on the task
- integrate back with the prover:

ENIGMA-style

The core idea

Learn to recognize and prefer for selection clauses that look like those that contributed to a proof in past successful runs.

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Possible Ways of Integrating the Learnt Advice

Priority:

- sort by model's Y/N and tiebreak by age
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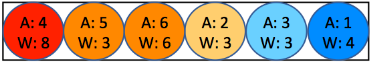
- 

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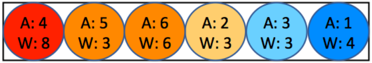
- 

Combine with the original strategy

$\mathcal{S} \oplus \mathcal{M}^{1,0}$: